NUMERICAL COMPUTATION OF INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM WITH SIMILARITY MEASURES

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Abstract- **In this paper, we focus on the solution of intuitionistic fuzzy assignment(IFA) problem, Similarity measures model also considered into account. Implement the consideration model to IFA problem. In this paper determining the IFA composite relative degree of similarity d_{ij}. The concept of the score function has also been used to find the composite relative similarity degree model. The numerical example are illustrated to proposed method to obtain the optimal solution**

Keywords: Intuitionistic fuzzy assignment problem, Intuitionistic Fuzzy number, Score function, Similarity measures of IFS.

I. INTRODUCTION

Assignment problem in operation research is used planetary in solving real world problems. An assignment problem is an main role in the assigning of persons to jobs or classes to rooms or operators to machines or drivers to trucks, trucks to routes or problems to research terms etc. Many researchers have worked on the concept of similarity measures of fuzzy sets. Chen M.S, investigate the similarity measures of fuzzy sets. Which are depends on the geometric model and set theoretic approach and matching function similarity measures of Intuitionistic fuzzy sets has been studied and developed by many authors chen M.S (1988) and Chen M.S (1955) introduced a matching function to calculate the degree of similarity between fuzzy sets. An Intuitionistic fuzzy set(IFS) A in X is defined as the following form, $A = \{x, \alpha_A(x), \beta_A(x) >/x\epsilon X\}$ where the functions $\alpha_A: X \to [0,1]$ and $\beta_A: X \to [0,1]$. Define the degree of membership and the degree of non-membership of element $x\epsilon X$ respectively and for every $x\epsilon X, 0 \le \alpha_A(x) + \beta_A(x) \le 1$; $\beta_A(x) = 1 - \alpha_A(x)$. The value of, $\pi_A(x) = 1 - \alpha_A(x) - \beta_A(x)$ is called the degree of non-determinacy or uncertainty of the element $x\epsilon X$ to the intuitionistic fuzzy set A.

II. PRELIMINARIES

Similarity measures of intuitionistic fuzzy set:

Let $S:\varphi(X)^2 \to [0,1]$, then the degree of similarity between A$\in \varphi(X)$ and B$\in \varphi(X)$ is defined as S(A,B) which satisfies the following conditions:

1. $0 \le S(A,B) \le 1$
2. S(A,B) = 1 iff A = B
3. S(A,B) = S(B,A)
4. S(A,C) ≤ S(A,B) iff S(A,C) ≤ S(B,C) if $A \le B \le C$

Similarity measures based on matching function:

Let $A \in \varphi(X)$ and $B \in \varphi(X)$, then the degree of similarity of A and B has been defined on matching function as,

$$S(A,B) = \frac{\sum_{j=1}^{n} \alpha_A(x_j)\alpha_B(x_j) + \beta_A(x_j)\beta_B(x_j) + \pi_A(x_j)\pi_{p_i}(x_j)}{\max\left\{\left(\alpha_A(x_j)^2 + \beta_A(x_j)^2 + \pi_A(x_j)^2\right), \left(\alpha_B(x_j)^2 + \beta_B(x_j)^2 + \pi_B(x_j)^2\right)\right\}}$$

Considering the weight Wj of each element xj $\in X$ we get,

$$S(A,B) = \frac{\sum_{j=1}^{n} W_j[\alpha_A(x_j)\alpha_B(x_j) + \beta_A(x_j)\beta_B(x_j) + \pi_A(x_j)\pi_{p_i}(x_j)]}{\max\left\{W_j\left(\alpha_A(x_j)^2 + \beta_A(x_j)^2 + \pi_A(x_j)^2\right), W_j\left(\alpha_B(x_j)^2 + \beta_B(x_j)^2 + \pi_B(x_j)^2\right)\right\}}$$

III. MATHEMATICAL MODELS BASED ON SIMILARITY MEASURES

Assume there are m persons and n jobs. Each job must be done by exactly one person to one job.

MODEL I: The crisp assignment problem, the cost C_{ij} is usually deterministic but real life situations it cannot be practical to know the exact values of these costs. In such an uncertain situations, instead of exact value of costs, we apply the form of composite relative degree (d_{ij}) of similarity to ideal solution, we can replace C_{ij} by d_{ij} in the crisp assignment problem we get,

$$\text{Max } Z = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} \quad (1)$$

Subject to,

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2 \ldots\ldots m$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad i = 1,2 \ldots\ldots m$$

$$x_{ij} = 0 \text{ or } 1 \qquad i,j = 1,2 \ldots\ldots m$$

MODEL II: The cost c_{ij} represent uncertain data like interval number,trapezoidal or triangular fuzzy or intuitionistic fuzzy number. In this chapter the cost has been considered to be IFN denoted by $\tilde{c_{ij}}$ which involves the positive and the negative evidence for thr membership of an element in a set. The cost of person i doing the job j is considered as an intuitionistic fuzzy number $\tilde{c_{ij}} = \{(\alpha_{ij}, \beta_{ij})\}$, i,j = 1,2………m. Here α_{ij} represents the degree of acceptance and β_{ij} represent the degree of rejection of the cost of doing jth job by the ith person. The problem is to find the composite relative degree d_{ij} to ideal solution denote ith person to jth job with the costs cij in the form of IFN.

Now we replace $\tilde{c_{ij}}$ by $\tilde{c_{ij}} = \{(\alpha_{ij}, \beta_{ij})\}$ then the crisp assignment problem becomes,

$$\text{Max } Z = \sum_{i=1}^{m}\sum_{j=1}^{m}\{(\alpha_{ij}, \beta_{ij})\}x_{ij} \quad (2)$$

The objective is to maximize acceptance degree α_{ij} and minimize the rejection degree β_{ij}. The objective function (2) can be written as,

$$\text{Max } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{ij}x_{ij}$$

$$\text{Max } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij}x_{ij}$$

Hence the intuitionistic fuzzy assignment problem becomes,

$$\text{Max } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{ij}x_{ij} \quad (3)$$

$$\text{Max } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij}x_{ij} \quad (4)$$

Subject to,

$$(\alpha_{ij} + \beta_{ij} - 1)\, x_{ij} \le 0 \quad (5)$$

$$\alpha_{ij}x_{ij} \ge \beta_{ij}x_{ij} \quad (6)$$

$$\beta_{ij}x_{ij} \ge 0 \quad (7)$$

MODEL III : The above multi objective model for the intuitionistic fuzzy assignment problem can be written as a single objective function is as follows :

$$\text{Max } Z = \sum_{i=1}^{m}\sum_{j=1}^{m}(\alpha_{ij} - \beta_{ij})x_{ij} \quad 8)$$

Subject to,

$$(\alpha_{ij} + \beta_{ij} - 1)\, x_{ij} \le 0$$

$$\alpha_{ij}x_{ij} \ge \beta_{ij}x_{ij}$$

$$\beta_{ij}x_{ij} \ge 0$$

SOLUTION PROCEDURE FOR FINDING OPTIMAL SOLUTION BY USING SIMILARITY MEASURES :

For an IFAP, Consider P = $\{p_1, p_1 \ldots\ldots p_m\}$ be a set of alternatives for a row or column in the cost matrix and consider c be an attribute describing the selection alternative. The characteristics of the alternative p_i are represented by IFS as,

$$p_i = \{(c, \alpha_{p_i}(c), \beta_{p_i}(c))\} \text{ , i =1,2…………m}$$

Where $\alpha_{p_i}(c)$ indicates the degree that the alternative p_i satisfies the attribute c and $\beta_{p_i}(c)$ indicates the degree that the alternative p_i does not satisfy the attribute c and $\alpha_{p_i} \epsilon [0,1]$ and $\beta_{p_i} \epsilon [0,1]$, $\alpha_{p_i}(c) + \beta_{p_i}(c) \le 1$.

Step 1 : Let $\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$ for all i = 1,2……… m find the positive ideal solution and negative ideal solution depends on IFN is defined as follows:

$$p^+ = \{\alpha_{p^+}(c), \beta_{p^+}(c)\} \quad (9)$$

$$p^- = \{c, \alpha_{p^-}(c), \beta_{p^-}(c)\} \quad (10)$$

Where,

$$\alpha_{p^+}(c) = \max_i\{\alpha_{p_i}(c)\} \text{ ,} \quad \beta_{p^+}(c) = \min_i\{\beta_{p_i}(c)\} \quad (11)$$

$$\alpha_{p^-}(c) = \min_i\{\alpha_{p_i}(c)\} \text{ ,} \quad \beta_{p^-}(c) = \max_i\{\beta_{p_i}(c)\} \quad (12)$$

Step 2 : Next we calculate the degree of similarity of the positive ideals intuitionistic fuzzy set p^+ and the alternative p_i and negative ideal intitionistic fuzzy set p^- and the alternative p_i by using the similarity measures of IFS's as follows :

$$s(p^+, p_i) = \frac{\alpha_{p^+}(c)\alpha_{p_i}(c) + \beta_{p^+}(c)\beta_{p_i}(c) + \pi_{p^+}(c)\pi_{p_i}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_i}(c)^2 + \beta_{p_i}(c)^2 + \pi_{p_i}(c)^2)\}} \quad (13)$$

$$s(p^-, p_i) = \frac{\alpha_{p^-}(c)\alpha_{p_i}(c) + \beta_{p^-}(c)\beta_{p_i}(c) + \pi_{p^-}(c)\pi_{p_i}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_i}(c)^2 + \beta_{p_i}(c)^2 + \pi_{p_i}(c)^2)\}} \quad (14)$$

For all i = 1,2…….m and j = 1,2……… m

Step 3: Next we calculate the relative similarity measures d_i based on (13),(14) corresponding to the alternatives Ai as follows :

$$d_i = \frac{s(p^+, p_i)}{s(p^+, p_i) + s(p^-, p_i)} \text{ , } i = 1,2 \ldots\ldots m \quad (15)$$

Step 4: continue step 1 to step 3 for remaining columns of the cost matrix and we find the relative similarity measure d_i corresponding to the alternative p_i for these columns. That is, the job with respect to the persons.

Step 5: Next we have to the form the matrix R_1 where $R_1 = [r_{ij}]_{m\times m}$ where r_{ij} is the relative similarity measures representing the jth person prefer to the ith job.

Step 6: we find the relative similarity measure d_j corresponding to the alternative p_i for these rows.The person with respect to the jobs and form the matrix R_2 where $R_2 = [s_{ij}]_{m\times m}$ where s_{ij} is the relative similarity measures representing the ith job is suitable for the jth person.

Step 7: Next we form the composite matrix $com(R_1 R_2) = (r_{ij}s_{ij})_{m\times m} = (d_{ij})_{m\times m}$. This matrix represent the composite relative degree of similarity of preference or suitability to ith job to the jth person or jth person is chosen the ith job.

Step 8: The composite matrix considered as a initial table of assignment problem in the maximization form (model 3) and using the Hungarian method to find the optimal assignment and the composite relative degree of similarity are maximizes.

NUMERICAL COMPUTATION:

The given IFAP with row representing three machines M1,M2,M3 and columns representing the three jobs J1,J2,J3 . The cost matrix are TIFN. To find the optimal assignment of jobs to machines.

jobs Machines	J1	J2	J3
M1	(0.3,0.4)	(0.5,0.1)	(0.4,0.1)
M2	(0.1,0,7)	(0.7,0.1)	(0.5,0.3)
M3	(0.6,0.2)	(0.2,0.4)	(0.3,0.2)

Solution :

Find the values of $s(p^+, M_j), s(p^-, M_j)$ and values of d_j in R_1 for machines with respect to jobs (column wise).In first column we find,

$$\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$$

i.e, $\pi_{p_1}(c) = 1 - \alpha_{p_1}(c) - \beta_{p_1}(c) = 1 - 0.3 - 0.4 = 0.3$

$\pi_{p_2}(c) = 1 - \alpha_{p_2}(c) - \beta_{p_2}(c) = 1 - 0.1 - 0.7 = 0.2$

$\pi_{p_3}(c) = 1 - \alpha_{p_3}(c) - \beta_{p_3}(c) = 1 - 0.6 - 0.2 = 0.2$

Here $\pi_{p^+}(c) = 0.3$ and $\pi_{p^-}(c) = 0.2$

$\alpha_{p^+}(c) = \max_i\{\alpha_{p_i}(c)\}$ =0.6,$\beta_{p^+}(c) = \min_i\{\beta_{p_i}(c)\}$ = 0.2

$\alpha_{p^-}(c) = \min_i\{\alpha_{p_i}(c)\} = 0.1$, $\beta_{p^-}(c) = \max_i\{\beta_{p_i}(c)\}$ = 0.7

$s(p^+, p_1)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_1}(c) + \beta_{p^+}(c)\beta_{p_1}(c) + \pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.6)(0.3) + (0.2)(0.4) + (0.3)(0.3)}{\max\{(0.6^2 + 0.2^2 + 0.3^2), (0.3^2 + 0.4^2 + 0.3^2)\}}$$

$$= \frac{0.35}{\max\{0.49, 0.34\}} = \frac{0.35}{0.49}$$

$s(p^+, p_1)$= 0.714

$s(p^-, p_1)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_1}(c) + \beta_{p^-}(c)\beta_{p_1}(c) + \pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.1)(0.3) + (0.7)(0.4) + (0.2)(0.3)}{\max\{(0.1^2 + 0.7^2 + 0.2^2), (0.3^2 + 0.4^2 + 0.3^2)\}}$$

$$= \frac{0.37}{\max\{0.54, 0.34\}} = \frac{0.37}{0.54}$$

$s(p^-, p_1)$=0.685

$$[r_{11}] = \frac{s(p^+, p_1)}{s(p^+, p_1) + s(p^-, p_1)}$$

$$= \frac{0.714}{0.714 + 0.685} = \frac{0.714}{1.399}$$

= 0.51

$s(p^+, p_2)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_2}(c) + \beta_{p^+}(c)\beta_{p_2}(c) + \pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.6)(0.1) + (0.2)(0.7) + (0.3)(0.2)}{\max\{(0.6^2 + 0.2^2 + 0.3^2), (0.1^2 + 0.7^2 + 0.2^2)\}}$$

$$= \frac{0.26}{\max\{0.49, 0.54\}} = \frac{0.26}{0.54}$$

$s(p^+, p_2)$= 0.481

$s(p^-, p_2)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_2}(c) + \beta_{p^-}(c)\beta_{p_2}(c) + \pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.1)(0.1) + (0.7)(0.7) + (0.2)(0.2)}{\max\{(0.1^2 + 0.7^2 + 0.2^2), (0.1^2 + 0.7^2 + 0.2^2)\}}$$

$$= \frac{0.54}{\max\{0.54, 0.54\}} = \frac{0.54}{0.54}$$

$s(p^-, p_2)$= 1

$$[r_{21}] = \frac{s(p^+, p_2)}{s(p^+, p_2) + s(p^-, p_2)}$$

$$= \frac{0.481}{0.481 + 1}$$

$$= \frac{0.481}{1.481}$$

= 0.325

$s(p^+, p_3)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_3}(c) + \beta_{p^+}(c)\beta_{p_3}(c) + \pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.6)(0.6) + (0.2)(0.2) + (0.3)(0.2)}{\max\{(0.6^2 + 0.2^2 + 0.3^2), (0.6^2 + 0.2^2 + 0.2^2)\}}$$

$$= \frac{0.46}{\max\{0.49, 0.44\}}$$

$$= \frac{0.46}{0.49}$$

$s(p^+, p_3)$= 0.939

$s(p^-, p_3)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_3}(c) + \beta_{p^-}(c)\beta_{p_3}(c) + \pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.1)(0.6) + (0.7)(0.2) + (0.2)(0.2)}{\max\{(0.1^2 + 0.7^2 + 0.2^2), (0.6^2 + 0.2^2 + 0.2^2)\}}$$

$= \frac{0.24}{\max\{0.54,0.44\}}$

$= \frac{0.24}{0.54}$

$s(p^-, p_3) = 0.444$

$$[r_{31}] = \frac{s(p^+, p_3)}{s(p^+, p_3) + s(p^-, p_3)} = \frac{0.939}{0.939 + 0.444} = \frac{0.939}{1.383} = 0.679$$

In second column we find,

$$\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$$

i.e, $\pi_{p_1}(c) = 1 - \alpha_{p_1}(c) - \beta_{p_1}(c) = 1 - 0.5 - 0.1 = 0.4$

$\pi_{p_2}(c) = 1 - \alpha_{p_2}(c) - \beta_{p_2}(c) = 1 - 0.7 - 0.1 = 0.2$

$\pi_{p_3}(c) = 1 - \alpha_{p_3}(c) - \beta_{p_3}(c) = 1 - 0.2 - 0.4 = 0.4$

Here $\pi_{p^+}(c) = 0.4$ and $\pi_{p^-}(c) = 0.2$

$\alpha_{p^+}(c) = \max_i\{\alpha_{p_i}(c)\} = 0.7$, $\beta_{p^+}(c) = \min_i\{\beta_{p_i}(c)\} = 0.1$

$\alpha_{p^-}(c) = \min_i\{\alpha_{p_i}(c)\} = 0.2$, $\beta_{p^-}(c) = \max_i\{\beta_{p_i}(c)\} = 0.4$

$s(p^+, p_1)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_1}(c) + \beta_{p^+}(c)\beta_{p_1}(c) + \pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.7)(0.5) + (0.1)(0.1) + (0.4)(0.4)}{\max\{(0.7^2 + 0.1^2 + 0.4^2), (0.5^2 + 0.1^2 + 0.4^2)\}}$$

$= \frac{0.52}{\max\{0.66,0.42\}}$

$= \frac{0.52}{0.66} = 0.788$

$s(p^-, p_1)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_1}(c) + \beta_{p^-}(c)\beta_{p_1}(c) + \pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.2)(0.5) + (0.4)(0.1) + (0.2)(0.4)}{\max\{(0.2^2 + 0.4^2 + 0.2^2), (0.5^2 + 0.1^2 + 0.4^2)\}}$$

$= \frac{0.22}{\max\{0.24,0.42\}} = \frac{0.22}{0.42}$

$= 0.524$

$$[r_{12}] = \frac{s(p^+, p_1)}{s(p^+, p_1) + s(p^-, p_1)} = \frac{0.788}{0.788 + 0.524} = \frac{0.788}{1.312}$$

$= 0.601$

$s(p^+, p_2)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_2}(c) + \beta_{p^+}(c)\beta_{p_2}(c) + \pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.7)(0.7) + (0.1)(0.1) + (0.4)(0.2)}{\max\{(0.7^2 + 0.1^2 + 0.4^2), (0.7^2 + 0.1^2 + 0.2^2)\}}$$

$= \frac{0.58}{\max\{0.66,0.54\}} = \frac{0.58}{0.66}$

$= 0.879$

$s(p^-, p_2)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_2}(c) + \beta_{p^-}(c)\beta_{p_2}(c) + \pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.2)(0.7) + (0.4)(0.1) + (0.2)(0.2)}{\max\{(0.2^2 + 0.4^2 + 0.2^2), (0.7^2 + 0.1^2 + 0.2^2)\}}$$

$= \frac{0.22}{\max\{0.24,0.54\}} = \frac{0.22}{0.54}$

$= 0.407$

$$[r_{22}] = \frac{s(p^+, p_2)}{s(p^+, p_2) + s(p^-, p_2)} = \frac{0.879}{0.879 + 0.407} = \frac{0.879}{1.286} = 0.684$$

$s(p^+, p_3)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_3}(c) + \beta_{p^+}(c)\beta_{p_3}(c) + \pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.7)(0.2) + (0.1)(0.4) + (0.4)(0.4)}{\max\{(0.7^2 + 0.1^2 + 0.4^2), (0.2^2 + 0.4^2 + 0.4^2)\}}$$

$= \frac{0.52}{\max\{0.66,0.36\}} = \frac{0.52}{0.66}$

$= 0.515$

$s(p^-, p_3)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_3}(c) + \beta_{p^-}(c)\beta_{p_3}(c) + \pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.2)(0.2) + (0.4)(0.4) + (0.2)(0.4)}{\max\{(0.2^2 + 0.4^2 + 0.2^2), (0.2^2 + 0.4^2 + 0.4^2)\}}$$

$= \frac{0.28}{\max\{0.24,0.36\}} = \frac{0.24}{0.36}$

$= 0.778$

$$[r_{32}] = \frac{s(p^+, p_3)}{s(p^+, p_3) + s(p^-, p_3)} = \frac{0.515}{0.515 + 0.778} = \frac{0.515}{1.286}$$

$= 0.398$

In third column we find,

$$\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$$

i.e, $\pi_{p_1}(c) = 1 - \alpha_{p_1}(c) - \beta_{p_1}(c) = 1 - 0.4 - 0.1 = 0.5$

$\pi_{p_2}(c) = 1 - \alpha_{p_2}(c) - \beta_{p_2}(c) = 1 - 0.5 - 0.3 = 0.2$

$\pi_{p_3}(c) = 1 - \alpha_{p_3}(c) - \beta_{p_3}(c) = 1 - 0.3 - 0.2 = 0.5$

Here $\pi_{p^+}(c) = 0.5$ and $\pi_{p^-}(c) = 0.2$

$\alpha_{p^+}(c) = \max_i\{\alpha_{p_i}(c)\} = 0.5, \beta_{p^+}(c) = \min_i\{\beta_{p_i}(c)\} = 0.1$

$\alpha_{p^-}(c) = \min_i\{\alpha_{p_i}(c)\} = 0.3$, $\beta_{p^-}(c) = \max_i\{\beta_{p_i}(c)\} = 0.3$

$s(p^+, p_1)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_1}(c) + \beta_{p^+}(c)\beta_{p_1}(c) + \pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.5)(0.4)+(0.1)(0.1)+(0.5)(0.5)}{\max\{(0.5^2+0.1^2+0.5^2),(0.4^2+0.1^2+0.5^2)\}}$$

$$= \frac{0.46}{\max\{0.51,0.42\}} = \frac{0.46}{0.51}$$

= 0.902

$s(p^-, p_1)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_1}(c)+\beta_{p^-}(c)\beta_{p_1}(c)+\pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_1}(c)^2+\beta_{p_1}(c)^2+\pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.3)(0.4)+(0.3)(0.1)+(0.2)(0.5)}{\max\{(0.3^2+0.3^2+0.2^2),(0.4^2+0.1^2+0.5^2)\}}$$

$$= \frac{0.25}{\max\{0.22,0.42\}} = \frac{0.25}{0.42}$$

= 0.595

$$[r_{13}] = \frac{s(p^+, p_1)}{s(p^+, p_1)+s(p^-, p_1)}$$

$$= \frac{0.902}{0.902+0.595} = \frac{0.902}{1.497}$$

=0.603

$s(p^+, p_2)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_2}(c)+\beta_{p^+}(c)\beta_{p_2}(c)+\pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_2}(c)^2+\beta_{p_2}(c)^2+\pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.5)(0.5)+(0.1)(0.3)+(0.5)(0.2)}{\max\{(0.5^2+0.1^2+0.5^2),(0.5^2+0.3^2+0.2^2)\}}$$

$$= \frac{0.38}{\max\{0.51,0.38\}} = \frac{0.38}{0.51}$$

= 0.745

$s(p^-, p_2)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_2}(c)+\beta_{p^-}(c)\beta_{p_2}(c)+\pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_2}(c)^2+\beta_{p_2}(c)^2+\pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.3)(0.5)+(0.3)(0.3)+(0.2)(0.2)}{\max\{(0.3^2+0.3^2+0.2^2),(0.5^2+0.3^2+0.2^2)\}}$$

$$= \frac{0.28}{\max\{0.22,0.38\}} = \frac{0.28}{0.38}$$

= 0.737

$$[r_{32}] = \frac{s(p^+, p_2)}{s(p^+, p_2)+s(p^-, p_2)}$$

$$= \frac{0.745}{0.745+0.737} = \frac{0.745}{1.482}$$

= 0.503

$s(p^+, p_3)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_3}(c)+\beta_{p^+}(c)\beta_{p_3}(c)+\pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_3}(c)^2+\beta_{p_3}(c)^2+\pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.5)(0.3)+(0.1)(0.2)+(0.5)(0.5)}{\max\{(0.5^2+0.1^2+0.5^2),(0.3^2+0.2^2+0.5^2)\}}$$

$$= \frac{0.46}{\max\{0.51,0.38\}} = \frac{0.46}{0.51}$$

= 0.824

$s(p^-, p_3)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_3}(c)+\beta_{p^-}(c)\beta_{p_3}(c)+\pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_3}(c)^2+\beta_{p_3}(c)^2+\pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.3)(0.3)+(0.3)(0.2)+(0.2)(0.5)}{\max\{(0.3^2+0.3^2+0.2^2),(0.3^2+0.2^2+0.5^2)\}}$$

$$= \frac{0.25}{\max\{0.22,0.38\}} = \frac{0.25}{0.38}$$

= 0.658

$$[r_{33}] = \frac{s(p^+, p_3)}{s(p^+, p_3)+s(p^-, p_3)}$$

$$= \frac{0.824}{0.824+0.658}$$

$$= \frac{0.824}{1.482}$$

= 0.556

Values of $s(p^+, M_J)$ for machines with respect to the jobs (column wise)

$s(p^+, M_j)$	J_1	J_2	J_3
M_1	0.714	0.788	0.902
M_2	0.481	0.879	0.745
M_3	0.939	0.515	0.824

Values of $s(p^-, M_J)$ for machines with respect to the jobs (column wise)

$s(p^-, M_j)$	J_1	J_2	J_3
M_1	0.685	0.524	0.595
M_2	1	0.407	0.737
M_3	0.444	0.778	0.658

Matrix R1 containing the values of dj for machines with respect to the jobs (column wise)

d_j or r_{ij}	J_1	J_2	J_3

M_1	0.51	0.601	0.603
M_2	0.325	0.684	0.503
M_3	0.679	0.398	0.556

Find the values of $s(p^+, J_i), s(p^-, J_i)$ and values of d_i in R_2 for jobs with respect to machines (row wise).

In first row we find,

$$\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$$

i.e, $\pi_{p_1}(c) = 1 - \alpha_{p_1}(c) - \beta_{p_1}(c) = 1 - 0.3 - 0.4 = 0.3$

$\pi_{p_2}(c) = 1 - \alpha_{p_2}(c) - \beta_{p_2}(c) = 1 - 0.5 - 0.1 = 0.4$

$\pi_{p_3}(c) = 1 - \alpha_{p_3}(c) - \beta_{p_3}(c) = 1 - 0.4 - 0.1 = 0.5$

Here $\pi_{p^+}(c) = 0.5$ and $\pi_{p^-}(c) = 0.3$

$\alpha_{p^+}(c) = \max_i \{\alpha_{p_i}(c)\}$ =0.5, $\beta_{p^+}(c) = \min_i \{\beta_{p_i}(c)\}$ = 0.1

$\alpha_{p^-}(c) = \min_i \{\alpha_{p_i}(c)\} = 0.3$, $\beta_{p^-}(c) = \max_i \{\beta_{p_i}(c)\} = 0.4$

$s(p^+, p_1)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_1}(c) + \beta_{p^+}(c)\beta_{p_1}(c) + \pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.5)(0.3) + (0.1)(0.4) + (0.5)(0.3)}{\max\{(0.5^2 + 0.1^2 + 0.5^2), (0.3^2 + 0.4^2 + 0.3^2)\}}$$

$$= \frac{0.34}{\max\{0.51, 0.34\}} = \frac{0.34}{0.51}$$

= 0.667

$s(p^-, p_1)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_1}(c) + \beta_{p^-}(c)\beta_{p_1}(c) + \pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$= \frac{(0.3)(0.3) + (0.4)(0.4) + (0.3)(0.3)}{\max\{(0.3^2 + 0.4^2 + 0.3^2), (0.3^2 + 0.4^2 + 0.3^2)\}}$$

$$= \frac{0.34}{\max\{0.34, 0.34\}} = \frac{0.34}{0.34}$$

= 1

$$[s_{11}] = \frac{s(p^+, p_1)}{s(p^+, p_1) + s(p^-, p_1)}$$

$$= \frac{0.667}{0.667 + 1} = \frac{0.667}{1.667} = 0.4$$

$s(p^+, p_2)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_2}(c) + \beta_{p^+}(c)\beta_{p_2}(c) + \pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.5)(0.5) + (0.1)(0.1) + (0.5)(0.4)}{\max\{(0.5^2 + 0.1^2 + 0.5^2), (0.5^2 + 0.1^2 + 0.4^2)\}}$$

$$= \frac{0.46}{\max\{0.51, 0.42\}} = \frac{0.46}{0.51}$$

= 0.902

$s(p^-, p_2)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_2}(c) + \beta_{p^-}(c)\beta_{p_2}(c) + \pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.3)(0.5) + (0.4)(0.1) + (0.3)(0.4)}{\max\{(0.3^2 + 0.4^2 + 0.3^2), (0.5^2 + 0.1^2 + 0.4^2)\}}$$

$$= \frac{0.31}{\max\{0.34, 0.42\}} = \frac{0.31}{0.42}$$

= 0.738

$$[s_{12}] = \frac{s(p^+, p_2)}{s(p^+, p_2) + s(p^-, p_2)}$$

$$= \frac{0.902}{0.902 + 0.738} = \frac{0.902}{1.64}$$

= 0.55

$s(p^+, p_3)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_3}(c) + \beta_{p^+}(c)\beta_{p_3}(c) + \pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.5)(0.4) + (0.1)(0.1) + (0.5)(0.5)}{\max\{(0.5^2 + 0.1^2 + 0.5^2), (0.4^2 + 0.1^2 + 0.5^2)\}}$$

$$= \frac{0.46}{\max\{0.51, 0.42\}} = \frac{0.46}{0.51}$$

= 0.902

$s(p^-, p_3)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_3}(c) + \beta_{p^-}(c)\beta_{p_3}(c) + \pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.3)(0.4) + (0.4)(0.1) + (0.3)(0.5)}{\max\{(0.3^2 + 0.4^2 + 0.3^2), (0.4^2 + 0.1^2 + 0.5^2)\}}$$

$$= \frac{0.26}{\max\{0.34, 0.42\}} = \frac{0.26}{0.42}$$

= 0.619

$$[s_{13}] = \frac{s(p^+, p_3)}{s(p^+, p_3) + s(p^-, p_3)}$$

$$= \frac{0.902}{0.902 + 0.619} = \frac{0.902}{1.521}$$

=0.593

In second row we find,

$$\pi_{p_i}(c) = 1 - \alpha_{p_i}(c) - \beta_{p_i}(c)$$

i.e, $\pi_{p_1}(c) = 1 - \alpha_{p_1}(c) - \beta_{p_1}(c) = 1 - 0.1 - 0.7 = 0.2$

$\pi_{p_2}(c) = 1 - \alpha_{p_2}(c) - \beta_{p_2}(c) = 1 - 0.7 - 0.1 = 0.2$

$\pi_{p_3}(c) = 1 - \alpha_{p_3}(c) - \beta_{p_3}(c) = 1 - 0.5 - 0.3 = 0.2$

Here $\pi_{p^+}(c) = 0.2$ and $\pi_{p^-}(c) = 0.2$

$\alpha_{p^+}(c) = \max_i \{\alpha_{p_i}(c)\}$ =0.7 , $\beta_{p^+}(c) = \min_i \{\beta_{p_i}(c)\}$ = 0.1

$\alpha_{p^-}(c) = \min_i \{\alpha_{p_i}(c)\} = 0.1$, $\beta_{p^-}(c) = \max_i \{\beta_{p_i}(c)\}$ = 0.7

$s(p^+, p_1)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_1}(c) + \beta_{p^+}(c)\beta_{p_1}(c) + \pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_1}(c)^2 + \beta_{p_1}(c)^2 + \pi_{p_1}(c)^2)\}}$$

$$=\frac{(0.7)(0.1)+(0.1)(0.7)+(0.2)(0.2)}{\max\{(0.7^2+0.1^2+0.2^2),(0.1^2+0.7^2+0.2^2)\}}$$
$$=\frac{0.18}{\max\{0.54,0.54\}}=\frac{0.18}{0.54}$$
= 0.333

$s(p^-,p_1)$
$$=\frac{\alpha_{p^-}(c)\alpha_{p_1}(c)+\beta_{p^-}(c)\beta_{p_1}(c)+\pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_1}(c)^2+\beta_{p_1}(c)^2+\pi_{p_1}(c)^2)\}}$$
$$=\frac{(0.1)(0.1)+(0.7)(0.7)+(0.2)(0.2)}{\max\{(0.1^2+0.7^2+0.2^2),(0.1^2+0.7^2+0.2^2)\}}$$
$$=\frac{0.54}{\max\{0.54,0.54\}}=\frac{0.54}{0.54}$$
= 1

$$[s_{21}]=\frac{s(p^+,p_1)}{s(p^+,p_1)+s(p^-,p_1)}$$
$$=\frac{0.333}{0.333+1}=\frac{0.333}{1.333}$$
= 0.250

$s(p^+,p_2)$
$$=\frac{\alpha_{p^+}(c)\alpha_{p_2}(c)+\beta_{p^+}(c)\beta_{p_2}(c)+\pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_2}(c)^2+\beta_{p_2}(c)^2+\pi_{p_2}(c)^2)\}}$$
$$=\frac{(0.7)(0.7)+(0.1)(0.1)+(0.2)(0.2)}{\max\{(0.7^2+0.1^2+0.2^2),(0.7^2+0.1^2+0.2^2)\}}$$
$$=\frac{0.54}{\max\{0.54,0.54\}}=\frac{0.54}{0.54}$$
= 1

$s(p^-,p_2)$
$$=\frac{\alpha_{p^-}(c)\alpha_{p_2}(c)+\beta_{p^-}(c)\beta_{p_2}(c)+\pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_2}(c)^2+\beta_{p_2}(c)^2+\pi_{p_2}(c)^2)\}}$$
$$=\frac{(0.1)(0.7)+(0.7)(0.1)+(0.2)(0.2)}{\max\{(0.1^2+0.7^2+0.2^2),(0.7^2+0.1^2+0.2^2)\}}$$
$$=\frac{0.18}{\max\{0.54,0.54\}}=\frac{0.18}{0.54}$$
= 0.333

$$[s_{22}]=\frac{s(p^+,p_2)}{s(p^+,p_2)+s(p^-,p_2)}$$
$$=\frac{1}{1+0.333}=\frac{1}{1.333}$$
= 0.750

$s(p^+,p_3)$
$$=\frac{\alpha_{p^+}(c)\alpha_{p_3}(c)+\beta_{p^+}(c)\beta_{p_3}(c)+\pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_3}(c)^2+\beta_{p_3}(c)^2+\pi_{p_3}(c)^2)\}}$$
$$=\frac{(0.1)(0.5)+(0.7)(0.3)+(0.2)(0.2)}{\max\{(0.1^2+0.7^2+0.2^2),(0.5^2+0.3^2+0.2^2)\}}$$
$$=\frac{0.42}{\max\{0.54,0.38\}}=\frac{0.42}{0.54}$$
= 0.778

$s(p^-,p_3)$
$$=\frac{\alpha_{p^-}(c)\alpha_{p_3}(c)+\beta_{p^-}(c)\beta_{p_3}(c)+\pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_3}(c)^2+\beta_{p_3}(c)^2+\pi_{p_3}(c)^2)\}}$$
$$=\frac{(0.1)(0.5)+(0.7)(0.3)+(0.2)(0.2)}{\max\{(0.1^2+0.7^2+0.2^2),(0.5^2+0.3^2+0.2^2)\}}$$
$$=\frac{0.3}{\max\{0.54,0.38\}}=\frac{0.3}{0.54}$$
= 0.556

$$[s_{23}]=\frac{s(p^+,p_3)}{s(p^+,p_3)+s(p^-,p_3)}$$
$$=\frac{0.778}{0.778+0.556}=\frac{0.778}{1.334}$$
=0.583

In third row we find,
$$\pi_{p_i}(c)=1-\alpha_{p_i}(c)-\beta_{p_i}(c)$$
i.e, $\pi_{p_1}(c)=1-\alpha_{p_1}(c)-\beta_{p_1}(c)=1-0.6-0.2=0.2$
$\pi_{p_2}(c)=1-\alpha_{p_2}(c)-\beta_{p_2}(c)=1-0.2-0.4=0.4$
$\pi_{p_3}(c)=1-\alpha_{p_3}(c)-\beta_{p_3}(c)=1-0.3-0.2=0.5$

Here $\pi_{p^+}(c)=0.5$ and $\pi_{p^-}(c)=0.2$

$\alpha_{p^+}(c)=\max_i\{\alpha_{p_i}(c)\}$ =0.6, $\beta_{p^+}(c)=\min_i\{\beta_{p_i}(c)\}$ = 0.2
$\alpha_{p^-}(c)=\min_i\{\alpha_{p_i}(c)\}=0.2$, $\beta_{p^-}(c)=\max_i\{\beta_{p_i}(c)\}$ = 0.4

$s(p^+,p_1)$
$$=\frac{\alpha_{p^+}(c)\alpha_{p_1}(c)+\beta_{p^+}(c)\beta_{p_1}(c)+\pi_{p^+}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_1}(c)^2+\beta_{p_1}(c)^2+\pi_{p_1}(c)^2)\}}$$
$$=\frac{(0.6)(0.6)+(0.2)(0.2)+(0.5)(0.4)}{\max\{(0.6^2+0.2^2+0.5^2),(0.6^2+0.2^2+0.4^2)\}}$$
$$=\frac{0.6}{\max\{0.65,0.56\}}=\frac{0.6}{0.65}$$
= 0.923

$s(p^-,p_1)$
$$=\frac{\alpha_{p^-}(c)\alpha_{p_1}(c)+\beta_{p^-}(c)\beta_{p_1}(c)+\pi_{p^-}(c)\pi_{p_1}(c)}{\max\{(\alpha_{p^-}(c)^2+\beta_{p^-}(c)^2+\pi_{p^-}(c)^2),(\alpha_{p_1}(c)^2+\beta_{p_1}(c)^2+\pi_{p_1}(c)^2)\}}$$
$$=\frac{(0.2)(0.6)+(0.4)(0.2)+(0.2)(0.4)}{\max\{(0.2^2+0.4^2+0.2^2),(0.6^2+0.2^2+0.4^2)\}}$$
$$=\frac{0.28}{\max\{0.24,0.56\}}=\frac{0.28}{0.56}$$
= 0.5

$$[s_{31}]=\frac{s(p^+,p_1)}{s(p^+,p_1)+s(p^-,p_1)}$$
$$=\frac{0.923}{0.923+0.5}=\frac{0.923}{1.423}$$
= 0.649

$s(p^+,p_2)$
$$=\frac{\alpha_{p^+}(c)\alpha_{p_2}(c)+\beta_{p^+}(c)\beta_{p_2}(c)+\pi_{p^+}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^+}(c)^2+\beta_{p^+}(c)^2+\pi_{p^+}(c)^2),(\alpha_{p_2}(c)^2+\beta_{p_2}(c)^2+\pi_{p_2}(c)^2)\}}$$
$$=\frac{(0.6)(0.2)+(0.2)(0.4)+(0.5)(0.4)}{\max\{(0.6^2+0.2^2+0.5^2),(0.2^2+0.4^2+0.4^2)\}}$$

$$= \frac{0.4}{\max\{0.65, 0.36\}} = \frac{0.4}{0.65}$$

= 0.615

$s(p^-, p_2)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_2}(c) + \beta_{p^-}(c)\beta_{p_2}(c) + \pi_{p^-}(c)\pi_{p_2}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_2}(c)^2 + \beta_{p_2}(c)^2 + \pi_{p_2}(c)^2)\}}$$

$$= \frac{(0.3)(0.5) + (0.4)(0.1) + (0.3)(0.4)}{\max\{(0.3^2 + 0.4^2 + 0.3^2), (0.5^2 + 0.1^2 + 0.4^2)\}}$$

$$= \frac{0.28}{\max\{0.65, 0.36\}} = \frac{0.28}{0.65} = 0.778$$

$$[s_{32}] = \frac{s(p^+, p_2)}{s(p^+, p_2) + s(p^-, p_2)} = \frac{0.615}{0.615 + 0.778} = \frac{0.615}{1.393}$$

= 0.441

$s(p^+, p_3)$

$$= \frac{\alpha_{p^+}(c)\alpha_{p_3}(c) + \beta_{p^+}(c)\beta_{p_3}(c) + \pi_{p^+}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^+}(c)^2 + \beta_{p^+}(c)^2 + \pi_{p^+}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.6)(0.3) + (0.2)(0.2) + (0.5)(0.5)}{\max\{(0.6^2 + 0.2^2 + 0.5^2), (0.3^2 + 0.2^2 + 0.5^2)\}}$$

$$= \frac{0.47}{\max\{0.65, 0.38\}} = \frac{0.47}{0.65}$$

= 0.723

$s(p^-, p_3)$

$$= \frac{\alpha_{p^-}(c)\alpha_{p_3}(c) + \beta_{p^-}(c)\beta_{p_3}(c) + \pi_{p^-}(c)\pi_{p_3}(c)}{\max\{(\alpha_{p^-}(c)^2 + \beta_{p^-}(c)^2 + \pi_{p^-}(c)^2), (\alpha_{p_3}(c)^2 + \beta_{p_3}(c)^2 + \pi_{p_3}(c)^2)\}}$$

$$= \frac{(0.2)(0.3) + (0.4)(0.4) + (0.2)(0.5)}{\max\{(0.2^2 + 0.4^2 + 0.2^2), (0.3^2 + 0.4^2 + 0.5^2)\}}$$

$$= \frac{0.32}{\max\{0.24, 0.5\}} = \frac{0.32}{0.5}$$

= 0.64

$$[s_{33}] = \frac{s(p^+, p_3)}{s(p^+, p_3) + s(p^-, p_3)} = \frac{0.723}{0.723 + 0.64} = \frac{0.723}{1.363} = 0.530$$

Values of $s(p^+, j_i)$ for jobs with respect to machines (row wise)

$s(p^+, j_i)$	J_1	J_2	J_3
M_1	0.667	0.902	0.902
M_2	0.333	1	0.778
M_3	0.923	0.615	0.723

Values of $s(p^-, j_i)$ for jobs with respect to machines (row wise)

$s(p^-, j_i)$	J_1	J_2	J_3
M_1	1	0.738	0.619
M_2	1	0.333	0.556
M_3	0.5	0.778	0.64

Matrix R2 containing the values of di for jobs with respect to the machines (row wise)

d_{ij} or s_{ij}	J_1	J_2	J_3
M_1	0.4	0.55	0.593
M_2	0.250	0.750	0.583
M_3	0.649	0.441	0.53

Next find the composite matrix representing the composite relative degree of similarity of jobs and machines. i.e., $com(R_1 R_2) = (r_{ij}s_{ij})_{m\times m} = (d_{ij})_{m\times m}$

$(r_{ij}s_{ij})_{m\times m}$	J_1	J_2	J_3
M_1	0.204	0.331	0.358
M_2	0.124	0.513	0.293
M_3	0.441	0.176	0.295

Hence the optimal assignment is,

$$M_1 \to J_3; M_2 \to J_2; M_3 \to J_1$$

Hence the optimal solution is

Z = 0.358 + 0.513 + 0.441 = 1.312

CONCLUSION

In this paper, Mathematical models of similarity measures are considered. Solution procedure of Intuitionistic fuzzy Assignment (IFA) problem with similarity measures are also discussed. Numerical example also illustrated to the proposed method. The result of the proposed method give the effective optimal solution to the solving IFA problem.

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